Coherent Tunneling and Instantons in Presence of Classical Chaos

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We consider the instanton approach to the problem of chaos assisted tunneling in the context of existing analytical and numerical results obtained in this field. We provide the estimation for the range of validity of this method and briefly discuss possible applications of chaos assisted tunneling for quantum computations.

§1. Introduction

The influence of classical chaos on dynamical tunneling1) in the semiclassical regime of quantum mechanics was observed in the pioneer work by Lin and Ballentine.2) Namely, it was detected in the numerical experiment that regions of chaotic motion in the classical phase space of the model system (periodically driven quartic double well) can enhance the rate of quantum tunneling between KAM-islands in several orders of magnitude compared to the ordinary tunneling in completely integrable case. The coherent nature of the tunneling was also emphasized (no leakage of the probability into the chaotic zone). The opposite impact of classical chaos on dynamical tunneling was found in the Ref.3) On the example of the model system with double-well potential affected by the periodic in time perturbation it was numerically demonstrated that for specific parameter values the tunneling between symmetry related KAM-islands can be coherently destructed i.e. the wave packet can be almost completely localized in one of the wells. For the theoretical explanation of the chaos assisted tunneling the multi-level model *) was proposed and tested for the two coupled quartic oscillators.4) The explanation of the tunneling rate enhancement observed in the numerical simulations is the replacement of usual two-level tunneling dynamics by more complicated three-level tunneling mechanism, appeared at the avoided level crossing of primarily regular (quasi-)energy doublet and chaotic eigenstate.5) Detailed description of numerical technique which can be used for simulation of chaos assisted tunneling can be found in the Ref.6) Possible applications of the chaos assisted tunneling are the active product selection in chemical reactions and the dynamical tunneling control for various physical systems.7)

The Refs.2),3),4),5),6),7) are clearly indicate the existing priority of numerical simulations for the investigation of the chaos assisted tunneling regime. A very few works devoted to the analytical investigation of this phenomenon do exist. Namely, the path integral formalism was primarily used for the analytical description of the

*) The three level-model is mainly exploited, since the singlet-doublet crossings are more likely to occur in the (quasi-)energy spectrum.

dynamical tunneling between symmetry related KAM-tori in the annular billiard possessing the mixed dynamics in the Ref.[8] Description of chaotic tunneling in terms of classical chaotic manifolds in the complex regime (complexified phase space) was done in the Ref.[9] Instanton technique,[10] primarily devised for quantum gauge field theories, can be used for more deep understanding of the chaos assisted tunneling regime in terms of path integral formulation of quantum mechanics. Particulary, chaotic instanton solutions, primarily obtained in the Ref.,[11] seem to play important role in dynamical tunneling through chaotic layer.[12]

The development of new analytical approaches (see also Refs[13,14]) is motivated by the following facts. Numerical simulations in the main are based on the Floquet theory.[15] One-dimensional systems affected by the monochromatic time-dependent perturbation (the aliquot frequencies do not change the picture) were mainly considered.[2,3,4,5,6,7] While the number of independent frequencies in the Fourier spectrum grows (or the number of degrees of freedom of the system increases) the size of the Floquet Hamiltonian matrix grows under the exponential law. Therefore, more computer resources are needed. In this case analytical methods including instanton approach may play an important role.

Another reason is based on the fact that numerical calculations can not give the complete picture, because it is impossible to cover in numerical simulations the whole continuous parametric region, the common features between different classes of systems can also escape one's attention. It does not underestimate the importance of numerical studies but rather states the necessity of analytical ones.

§2. Instanton approach to chaos assisted tunneling

The instanton method showed oneself well in description of the spectral properties and tunneling probabilities between lower eigenstates.[16] The attempt to generalize the instanton method for non-autonomous Hamilton systems was made in the Refs.[11,12] In the present work we estimate the parametric region where calculations made in Refs.[11,12] are valid. For distinctness we consider the system with the Euclidean Hamiltonian $H = p^2/2 - \omega_0^2 \cos x + \epsilon x \cos \nu t$. Here the coordinate x varies from $-\infty$ to $+\infty$ and t is real and denotes Euclidian time. This Hamiltonian differs from one used in Refs.[11,12] by the form of time-dependent perturbation. The results obtained in these papers remain valid for this case up to the numerical factor. It emphasizes their applicability for a wide class of the systems with periodic in space potentials (single potential well in each period) affected by the small periodic in time perturbation. The reason is the universality of the separatrix destruction mechanism.[17] Limitations to the range of validity of the particular realization of the instanton method and the results obtained in Refs.[11,12] come from the following assumptions made. Stochastic layer is narrow and homogeneous, therefore the stability island near the elliptic point of the first resonance has to be indistinguishable. Euclidian actions of the perturbed (chaotic) single-instanton configurations approximately equal the Euclidian actions of non-perturbed instantons

at non-minimal energies (see Ref.[12]). These conditions lead to the restrictions:[17]

$$\bar{\epsilon} \ll \alpha \ll 1, \quad \nu > \omega_0, \quad \alpha = \frac{I}{\omega(I)} \left| \frac{d\omega}{dI} \right| . \tag{2·1}$$

Here $\bar{\epsilon} \equiv \epsilon/\omega_0^2$ denotes the dimensionless coupling constant, and α is the parameter of nonlinearity,[17] I is the action variable, $\omega(I)$ is the frequency of nonlinear oscillations.

For indistinguishability of the first resonance additional restrictions are needed. The first resonance width in frequency estimated by means of the standard technique[17] is given by the expression:

$$\delta\omega \sim (\epsilon\omega_1')^{\frac{1}{2}}, \quad \omega_1' \equiv \frac{d\omega(I)}{dI} |_{I=I_1} . \tag{2·2}$$

The value I_1 is determined by the resonance condition $\omega(I_1) = \nu$. Therefore the first resonance width in action is estimated as $\delta I = (\epsilon/\omega_1')^{\frac{1}{2}}$. The condition for indistinguishability of the first resonance is $\delta I \ll \omega_0$. Thus the system independent restriction has the form:

$$\bar{\epsilon}^{\frac{1}{2}} \ll 1. \tag{2·3}$$

The conditions (2·1, 2·3) define the parametric range for the instanton technique used in the Refs.[11),12] to be valid.

Let us discuss the prospects for chaos assisted tunneling to be applied for the enhancement of the stability of quantum computations and reliability of quantum computers. Quantum computer consists of qubits (quantum bits) which states superposition encodes the initial and final data. Quantum computation operations are realized by the unitary operators (consisted of the elementary operations – quantum gates) acting on these superimposed states. There are two main obstacles for implementation of quantum computations in practice. One of them is the problem of decoherence,[18] which means the superimposed state to be destructed rapidly due to unavoidable interaction with the environment, and to be converted into the mixture of the qubit states. It destructs the quantum computation process. Another problem is the existence of small unknown or uncontrollable residual interaction among qubits.[19] Assume that the decoherence process is slow enough (because of some reasons which are not discussed here) and does not break down quantum computations. Unitary evolution operator has the form $U = Exp(iH)$, where H is the Hamiltonian, which is known if one wishes to receive the definite result. However, residual interactions between qubits lead the Hamiltonian to be consisted of two parts $H = H_0 + V$. The first part H_0 is "large" in the sense it is mainly determine quantum evolution. The second part V plays the role of small unknown perturbation originated from the uncontrollable interaction between qubits. For reliability of results obtained on quantum computer quantum calculations have to be stable with respect to these perturbations. Quantitative parameter called fidelity determining the stability of computational process was introduced and widely exploited.[20] The residual interaction between qubits can originate from the tunneling transitions between them. Thus it is important to suppress such undesirable tunneling coupling. It can be achieved in chaos assisted tunneling regime.[3] Therefore the investigation of the destructive interference of chaotic instanton contributions is of importance.

§3. Conclusion

We have demonstrated the necessity of analytical approaches to the tunneling problem. Application of the instanton method to the problem of chaos assisted tunneling was discussed in the context of existing results. Chaotic instanton solutions[11),12)] may be useful particulary in investigation of chaos assisted tunneling in one-dimensional systems with degenerate classical ground state affected by non-monochromatic time-dependent perturbation (having two or more independent frequencies in its Fourier spectrum). Consideration of such systems seems attractive, since it was demonstrated that the control of dynamical tunneling can be achieved in bichromatically driven pendulum.[7)]

We provided the estimation for the range of validity of perturbed instanton approach for a wide class of one-dimensional systems with periodic potential affected by periodic in time perturbation. Corresponding conditions for the system parameters were formulated. Nevertheless for further justification and development of the method the comparison with numerical data and existing analytical methods is needed. Possible application of chaos assisted tunneling in connection with quantum computer stabilization was also briefly discussed.

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